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1978 J. Phys. A: Math. Gen. 11 L179

(http://iopscience.iop.org/0305-4470/11/8/003)

J. Phys. A: Math. Gen., Vol. 11, No. 8, 1978. Printed in Great Britain.

LETTER TO THE EDITOR

On the time-asymptotic particle interpretation in the Friedman–de Sitter space

G Schäfer
Fachbereich Physik der Universität Konstanz, Postfach 7733, D-7750 Konstanz, W Germany

Received 19 May 1978, in final form 20 June 1978

Abstract. It is shown that in that closed Friedman universe which is the de Sitter space the time-asymptotic quasi-classical particle concept, as understood in this paper, is not possible for the conformally coupled Klein–Gordon field. It is argued that for the well known infinite rate of particle creation per unit volume the particles do not belong to physical modes.

1. Introduction

Up to now it has not been explicitly excluded but has even sometimes been stated that in the de Sitter space a time-asymptotic quasiclassical particle concept is possible notwithstanding the fact that the particle creation rate per unit volume calculated with the supposed particle concept is infinite, which implies that physically either the particle concept or the de Sitter space is inadequate. In this paper it will be shown that indeed the particle concept applied is unphysical and furthermore that a time-asymptotic physical particle concept isn't possible at all. On the other hand, for the construction of a meaningful Fock representation on the de Sitter space, a curvature-asymptotic (vanishing curvature scalar) covariant 'particle' concept (for the conformally coupled Klein–Gordon field and large quantum numbers these modes become quasi-classical†) which doesn't give rise to a creation rate, has proved to be useful. A physical particle interpretation of all of these modes however does not seem feasible.

For the statements above consider the papers of Gutzwiller (1956), Rumpf (1976a, b) and Dowker and Critchley (1976) where the time-asymptotic quasiclassical particle concept in the de Sitter space is either used or at least mentioned. Compare also the papers of Fulling *et al* (1974), Parker and Fulling (1974) and Parker (1975) with the adiabatic particle concept (generalised WKB concept), the papers of Woodhouse (1976, 1977) and Audretsch and Schäfer (1978) with other sorts of WKB particle concepts, and the papers of Grib *et al* (1976), Mamayev *et al* (1976) and Schäfer and Dehnen (1977) with the particle concept based on Hamiltonian diagonalisation, which all get time-asymptotic particle modes in the de Sitter space. Also the papers of Nachtmann (1967), Chernikov and Tagirov (1968), Tagirov (1973), Candelas and Raine (1975) and Dowker and Critchley (1976) with the curvature-asymptotic 'particle' concept do not explicitly exclude time-asymptotic particle modes. Only the

† 'Quasi-classical' means 'quasiclassical' in the sense of this paper or the papers of Chernikov and Tagirov.

0305-4770/78/0008-9179$01.00

paper of Dowker and Critchley (1976) includes a hint that 'time-asymptotic particles' are a contradictory concept.

For the concept of observer-dependent particles in the de Sitter space and its physical implications the papers of Gibbons and Hawking (1977) and Hajicek (1977) are illuminating.

Notation: $c = \hbar = 1$, Metric signature $(+---)$, $g \equiv \det(g_{\mu\nu})$, $\Sigma \equiv$ space-like hypersurface (Einstein universe), $d\Sigma_\mu \equiv$ oriented coordinate volume element on Σ, $\partial_\mu \equiv \partial/\partial x^\mu$, indices are omitted sometimes.

2. De Sitter space

The de Sitter space may be given by the following; the 'entire' space covering coordinate system $(\eta, \chi, \theta, \phi)$ with Robertson–Walker metric

$$ds^2 = \frac{a^2}{\cos^2 \eta}(d\eta^2 - d\sigma^2) \tag{1}$$

where

$$d\sigma^2 = d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta \, d\phi^2), \qquad 0 < \chi, \quad \theta < \pi, \quad 0 \leq \phi < 2\pi,$$

is a line element for the unit Einstein universe, $a =$ constant, and $-\pi/2 < \eta < \pi/2$. The curvature scalar R has the constant value $12/a^2$.

3. Klein–Gordon field

As the field equation for the complex-valued massive scalar field Φ we take the conformally coupled one

$$(\Box + R/6 + m^2)\Phi = 0, \qquad \Box = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu). \tag{2}$$

The hypersurface-independent scalar product according to which the Φ-functions have to be normalised may be written

$$(\Phi_1, \Phi_2) = \frac{i}{2m} \int_\Sigma \Phi_1^* \overleftrightarrow{f^\mu} \Phi_2 \, d\Sigma_\mu, \qquad \overleftrightarrow{f^\mu} = \sqrt{-g} g^{\mu\nu} \overrightarrow{\partial}_\nu - \overleftarrow{\partial}_\nu g^{\nu\mu} \sqrt{-g}.$$

With the help of the decomposition $\Phi = Z \exp(iW)$, Z and W real, we get from equation (2), after separating the real and the imaginary parts, the equations

$$\frac{\Box Z}{Z} + \frac{R}{6} = g^{\mu\nu}(\partial_\mu W)(\partial_\nu W) - m^2 \tag{2a}$$

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} Z^2 g^{\mu\nu} \partial_\nu W) = 0 \tag{2b}$$

and from equation (3) or (2b) the equation

$$(\Phi, \Phi) = -\frac{1}{m} \int_\Sigma \sqrt{-g} Z^2 g^{\mu\nu} \partial_\nu W \, d\Sigma_\mu. \tag{3a}$$

With regard to the equations ($5a$, b) we are forced to choose this form of the equations ($2a$, b) respectively, i.e. equation ($5a$) holds after dividing through by Ze^{iW} and equation ($5b$) after dividing through by e^{iW} and multiplying by Z.

In the metric (1) equation (2) or the equations ($2a$, b) are separable and we can write, if we take into consideration the time-independence of equation ($3a$),

$$\Phi = \frac{\cos\eta}{a}\frac{1}{\sqrt{W_0'(\eta)}}e^{iW_0(\eta)}\Lambda_n(\chi,\theta,\phi) \tag{4}$$

with W_0 real and $W_0' = dW_0/d\eta$. The Λ_n-functions set up a *complete* system of functions on the space-like hypersurfaces $\eta = \text{constant}$, n is a discrete separation constant and W_0 depends on η only.

To discuss the possibility of interpreting the Φ's of equation (4) as quasi-classical particle solutions we quote both the classical equations for a swarm of freely moving point particles:

$$g^{\mu\nu}(\partial_\mu S)(\partial_\nu S) - m^2 = 0 \qquad \text{Hamilton–Jacobi equation} \tag{5a}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\rho g^{\mu\nu}\partial_\nu S) = 0 \qquad \text{equation of continuity.} \tag{5b}$$

S (real) is the action and ρ (real, $\geqslant 0$) the density distribution of the particles.

From equation ($5b$) it follows that the expression

$$-\frac{1}{m}\int_\Sigma \sqrt{-g}\rho g^{\mu\nu}\partial_\nu S\, d\Sigma_\mu \tag{6}$$

is constant in time. Because equations ($5a$) and ($5b$) are also separable in the metric (1) we can write, using the time-independence of expression (6),

$$S = S_0(\eta) + S_1(\chi,\theta,\phi) \tag{7a}$$

$$\rho = \rho_0(\eta)\rho_n(\chi,\theta,\phi) \qquad \text{with} \quad \rho_0 = \left(\frac{\cos\eta}{a}\right)^2\frac{1}{S_0'}. \tag{7b}$$

In building up the WKB expression $\sqrt{\rho}\, e^{iS}$ it follows, with the help of equations ($7a$, b), that

$$\sqrt{\rho}\, e^{iS} = \frac{\cos\eta}{a}\frac{1}{\sqrt{S_0'(\eta)}}e^{iS_0(\eta)}N_\varkappa(\chi,\theta,\phi) \tag{8}$$

with $N_\kappa = \sqrt{\rho_1}\, e^{iS_1}$ and κ a real separation constant. From equations ($5a$, b) we get an equation for S_0,

$$\frac{\cos^2\eta}{a^2}[S_0'^2 - \kappa^2] - m^2 = 0, \qquad 0 \leqslant \kappa, \tag{9}$$

and an equation for W_0 from equations ($2a$, b),

$$\frac{\cos^2\eta}{a^2}\left[\left(\frac{1}{2}\frac{W_0'''}{W_0'^3} - \frac{3}{4}\frac{W_0''^2}{W_0'^4} + 1\right)W_0'^2 - n^2\right] - m^2 = 0, \qquad n = 1, 2, 3, \ldots. \tag{10}$$

Our *condition* that the Φ-functions of equation (4) describe time-asymptotic freely moving quasi-classical particles will, in comparing equation (10) with equation (9), be

$$\frac{\cos^2\eta}{a^2}W_0'^2\left[\frac{1}{2}\frac{W_0'''}{W_0'^3}-\frac{3}{4}\frac{W_0''^2}{W_0'^4}\right]\to 0 \qquad \text{for} \quad |\eta|\to\frac{\pi}{2}. \tag{11}$$

(In the limit $|\eta|\to\pi/2$ the difference between κ and n is empty.) Incorporating into expression (4) the asymptotic behaviour (11) and using equation (10) again we write

$$\frac{1}{\sqrt{W_0'}}e^{iW_0}=\frac{1}{\sqrt{w(\eta)}}\Gamma(\tau) \quad \text{with } \tau=\int^{\eta}\omega\,d\eta$$

and

$$\omega=\left(\frac{m^2a^2}{\cos^2\eta}+n^2\right)^{1/2}$$

and get for Γ, applying equation (2), the 'first' (besides an irrelevant factor for $|\eta|\to\pi/2$) Chakraborty equation (Chakraborty 1973, equation (3.3))

$$\frac{\cos^2\eta}{a^2}\omega^2\left[\frac{d^2\Gamma}{d\tau^2}+(1+\epsilon_2)\Gamma\right]=0, \qquad \epsilon_2=\frac{3}{4}\left(\frac{\omega^1}{\omega^2}\right)^2-\frac{1}{2}\frac{\omega''}{\omega^3}. \tag{12}$$

Now the condition (11) essentially reads

$$\epsilon_2\to 0 \qquad \text{for } |\eta|\to\pi/2. \tag{13}$$

But in calculating ϵ_2 for finite n we get

$$\epsilon_2\to -1/4a^2m^2 \qquad \text{for} \quad |\eta|\to\pi/2. \tag{14}$$

This however means that asymptotically the effective mass of the Klein–Gordon field 'at rest' becomes smaller than m, i.e. self-interactions of the Klein–Gordon field which are caused by the gravitational field remain. Therefore time-asymptotic quasi-classical free (physical) particles are not possible†.

4. Discussion

If one looks closer at the ϵ_2 term in expression (14) it is revealed that asymptotically, for finite n, ϵ_2 is proportional to $\hbar^2$. Therefore in the WKB approximation of Woodhouse (1976, 1977) ϵ_2 doesn't count‡ and so it must be concluded (n infinite doesn't cause trouble) that time-asymptotic quasiclassical particle modes exist. Audretsch and Schäfer (1978) get particles in the de Sitter space because the vanishing of $\rho_0(\eta)$ from equation (7*b*) for $|\eta|\to\pi/2$ deletes their ϵ_2 term. For Fulling *et al* (1974), Parker and Fulling (1974) and Parker (1975) because $\epsilon_4\to 0$ (Chakraborty's terminology) for $|\eta|\to\pi/2$ asymptotic particle modes exist. Gutzwiller (1956), Rumpf (1976*a*, *b*) and (with reservations) Dowker and Critchley (1976) who ask (time-asymptotically) for time exponentials receive particles too because $\epsilon_2\to$ constant for $|\eta|\to\pi/2$ (cf. (14)). And Grib *et al* (1976), Mamayev *et al* (1976) and Schäfer and Dehnen (1977) who define particles through diagonalisation of the Hamiltonian obviously get particle modes.

† With the same argument, yet applied to quantum number-asymptotics, Chernikov and Tagirov (1968) have excluded the minimally coupled Klein–Gordon equation as the correct one for the massive scalar field.
‡ If $ma \gg 1$.

Concerning the Feynman–Schwinger–DeWitt approach used by Candelas and Raine (1975) and Dowker and Critchley (1976) it should be stated that it works as well (without creation rate) for the minimally coupled Klein–Gordon field where no quantum number-asymptotic (correspondence principle) quasi-classical particle modes exist so that in the Feynman–Schwinger–DeWitt approach further physical arguments must be imposed as, in another context, Candelas and Raine (1977) have pointed out explicitly. Our 'strongest' particle concept above supplies such an argument. This concept excludes, for example, at the beginning of the expansion (special expansion law), the interpretation of the boundary conditions imposed in the Feynman–Schwinger–DeWitt formalism by Chitre and Hartle (1977) in terms of particles and seems to explain their ultra-relativistic result in contrast to Audretsch's and Schäfer's (1978) nonrelativistic one (for the expansion law discussed by Audretsch and Schäfer (1978) their particle concept coincides with that advocated in this paper). Also the particle concept used by Gibbons (1975) when investigating particle creation in plane-wave space times is the same as that in this paper. His results concerning these particles are meaningful.

Acknowledgment

For stimulating discussion I wish to thank J Audretsch.

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